## HEAT TRANSFER COEFFICIENT IN TURBULENT FLOW

OF HEAT-RELEASING LIQUID

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In Kutateladze's well-known monograph [1] the expression

$$
\begin{equation*}
\frac{\alpha}{\alpha_{0}}=\frac{1}{1+A Z}, \quad Z=\frac{d q_{v}}{4 q_{w}} \tag{1}
\end{equation*}
$$

was obtained analytically for the correction factor allowing for internal heat release or heat absorption in calculation of the heat transfer coefficient in a round tube.

Here $Z$ is the relative density of the internal heat source. The constant coefficient A is expressed in the following way in terms of quadratures of the distributions of the dimensionless velocity $\omega(\xi)=\omega /\langle\omega\rangle$ and the turbulent thermal conductivity $\lambda_{t}(\xi)$ over the radius $\xi=2 \mathrm{r} / \mathrm{d}$

$$
\begin{gather*}
A=\left\{\int_{0}^{1} \omega \xi\left[\int_{\xi}^{1}\left(\frac{\Omega}{\xi}-1\right) \frac{d \xi}{1+\lambda_{i} / \lambda}\right] d \xi\right\}\left\{\int_{0}^{1} \omega \xi\left[\int_{\xi}^{1} \frac{\Omega}{\xi} \frac{d \xi}{1+\lambda_{t} / \lambda}\right] d \xi\right\}^{-1} \\
\Omega(\xi)=\int_{0}^{\xi} \omega \xi d \xi \tag{2}
\end{gather*}
$$

Numerical values of the coefficient A were obtained in [1] for special cases of laminar flow with a parabolic velocity profile ( $\mathrm{A}=0.272$ ) and turbulent flow with a velocity distribution conforming to a $1 / 7$ law and Prandtl number $\mathrm{P}=0(\mathrm{~A}=0.0834)$.

We give the results of calculation of this coefficient for the case of a turbulent flow with $\mathrm{P} \neq 0$.
Integrating by parts in the numerator and denominator of Eq. (2) we obtain

$$
\begin{equation*}
A=1-\frac{1}{2}\left[\int_{0}^{1} \frac{\xi \Omega}{1+\lambda_{t} / \lambda} d \xi\right]\left[\int_{0}^{1} \frac{\Omega^{2}}{\left(1+\lambda_{i} / \lambda\right) \xi} d \xi\right]^{-1} \tag{3}
\end{equation*}
$$



Fig. 1

The main difficulty in reducing Eq. (3) to a form convenient for numerical calculations lies in the choice of an approximating relationship giving the distribution of $\lambda_{t} / \lambda$ over the tube radius. It should be noted that

$$
\begin{equation*}
\frac{\lambda_{t}}{\lambda}=\frac{P}{P_{t}} \frac{\mu_{t}}{\mu} \tag{4}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{t}}$ is the turbulent Prandtl number.

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We use the Prandtl expression for the turbulent tangential stress in terms of the mixing length $l$ and also its representation in terms of the turbulent viscosity coefficient $\mu_{\mathrm{t}}$

$$
\begin{equation*}
\tau_{t}=\rho\left(l \frac{d w}{d r}\right)^{2}, \quad \tau_{t}=-\mu_{t} \frac{d w}{d r} \tag{5}
\end{equation*}
$$

By comparison we have

$$
\begin{equation*}
\frac{\mu_{t}}{\mu}=-\frac{\rho}{\mu} l^{2} \frac{d w}{d r} \tag{6}
\end{equation*}
$$

According to I. Nikuradze's measurements, at $\mathrm{R}=\rho\left\langle\omega>\mathrm{d} / \mu>10^{5}\right.$ the distribution of the mixing length over the tube radius is independent of $R$. The well-known interpolation formula [1] gives in this case

$$
\begin{equation*}
2 l / d=0.14-0.08 \xi^{2}-0.06 \xi^{4} \tag{7}
\end{equation*}
$$

It is assumed in what follows that the velocity distribution over the cross section of the tube conforms to a $1 / 7$ law

$$
\begin{equation*}
\omega=w /\langle w\rangle=1.22(1-\xi)^{1 / 7} \tag{8}
\end{equation*}
$$

Using (4), (6), (7), and (8) and performing the necessary transformations we put Eq. (3) in a form suitable for numerical calculations on an electronic digital computer. The results of these calculations are given in the form of a relationship between the coefficient $A$ and the criterion $R^{*}=R P / P_{t}$ in Fig. 1 . It should be remembered that these results are valid for $R>10^{5}$.

## LITERATURE CITED

1. S. S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Mashgiz, Moscow-Leningrad, 1962.
